Global sensitivity analysis in integrated assessment modeling

Alena Miftakhova

alena.miftakhova@business.uzh.ch

University of Zurich[UZH]

Dept. of Business Administration

Quantitative Business Administration

Abstract

One key question in IAM is how sensitive the climate policy inferences produced by models are to the uncertainty in their initial assumptions and the results of calibrations. We argue for including structured global sensitivity analysis (GSA) as a standard, indispensable step of the modeling practice in climate economics. We apply a high-efficiency GSA method based on polynomial chaos expansions to DICE.

Our analysis provides two key insights. First, a selection of a subset of parameters of interest might overlook the most influential ones. Second, the opposite strategy—pooling them all together and considering the model as a "black box"—distorts significance indices. The best practice therefore is to consider all exogenous parameters for an analysis but ensure that the fundamental relations within model's structure are respected.

Background

Because the inference from IAMs often implies climate policy advice, it is crucial to inform policymakers about the *form, magnitude, and sources of associated uncertainty*.

Local methods, which explore the sensitivity of a model at particular points in the parameter space, are popular thanks to their simplicity. They, however, suffer from dependence on the points chosen for analysis, and might thus overlook important regions of interest.

Global methods offer a comprehensive picture of uncertainty in model's output by, first, exploring the entire parameter space and, second, accounting for interactions and nonlinearities in the effects of parameters on output—the advantages that come at a high computational cost.

The existing literature on GSA applied to DICE suggests that the most influential parameters are not necessarily those subjectively selected by experts [1]. It also shows that interaction effects, usually ignored, are not negligible.

PCE-based GSA Method

We focus on a GSA method [5] that uses *polynomial chaos expansions* (PCE) to produce a full variance-based representation of uncertainty in a model much more efficiently than alternative techniques do.

1. **Problem definition.** The model of interest is viewed as a *mapping* from parameters space, Θ, onto output quantity Y,
$$Y = \mathcal{M}(\boldsymbol{\Theta}),$$
where $\mathcal{M} : \boldsymbol{\Theta} \mapsto Y$, with $\boldsymbol{\Theta} \in \mathbb{R}^M$ and $Y \in \mathbb{R}$. A *probability density function* f_θ is assigned to each parameter of interest θ.
2. **Construction of a meta-model.** Given a sample of model runs, the output is approximated with a sum of multivariate orthogonal polynomials of parameter values,
$$Y = \sum_{\alpha \in \mathbb{N}^M} y_\alpha \Psi_\alpha(\Theta).$$
3. **Variance decomposition.** The measures of sensitivity, *Sobol' indices*, are the shares of total variance attributed to subsets of parameters $\mathbf{u} \subset \{1, \cdots, M\}$,
$$S_{\mathbf{u}} = \frac{\text{Var}\left[\mathcal{M}_{\mathbf{u}}(\boldsymbol{\Theta}_{\mathbf{u}})\right]}{\text{Var}[Y]}.$$
Total index reflects the total contribution of a parameter; *first order index*, it's individual contribution; *higher order indices*, importance of interaction with other parameters.
4. **PCE approximation, once obtained, provides**
   (a) ready-to-use *meta-model* of the original model,
   (b) *moments* of output quantity,
$$\mu_Y^{PC} = \mathbb{E}\left[\mathcal{M}^{PC}(\boldsymbol{\Theta})\right] = y_0,$$
$$\text{Var}^{PC}[Y] = \mathbb{E}\left[\left(\mathcal{M}^{PC}(\boldsymbol{\Theta}) - \mu_Y^{PC}\right)^2\right] = \sum_{\alpha \in \mathcal{A}, \alpha \neq 0} y_\alpha^2,$$
   (c) *full set of Sobol' indices* computed analytically:
$$S_{\mathbf{u}}^{PC} = \frac{\sum_{\alpha \in \mathcal{A}_{\mathbf{u}}} \hat{y}_\alpha^2}{\text{Var}^{PC}[\hat{Y}]}.$$

Implementation: We use UQLab toolbox for Matlab [2] to apply PCE-based GSA.

GSA applied to DICE

We apply the method to DICE2007 and select *Social Cost of Carbon in 2005* as an output of interest.

The results of the analysis of **8 selected parameters** (Figure 1) are consistent with [3]: damage function coefficient contributes to the variance the most, followed by climate sensitivity and growth rate of technology.

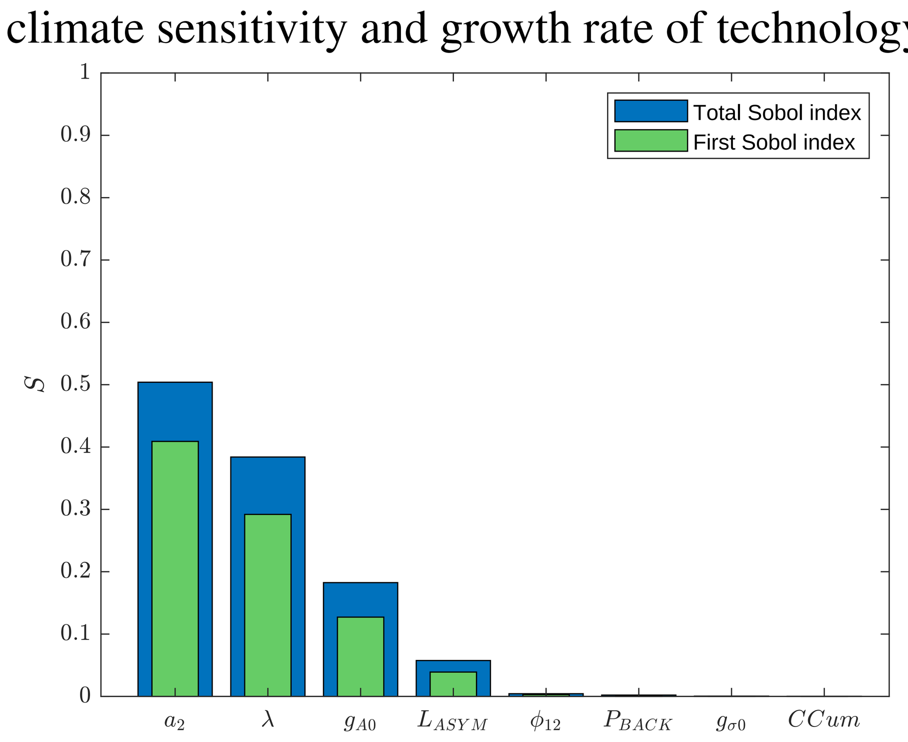

Figure 1: Total and first Sobol indices for 8 parameters.

The analysis of **all 51 parameters** in Figure 2 demonstrates, however, that the parameter selection above is highly subjective and omits the important ones—the exponent of the damage function, capital elasticity, and elasticity of marginal utility of consumption.

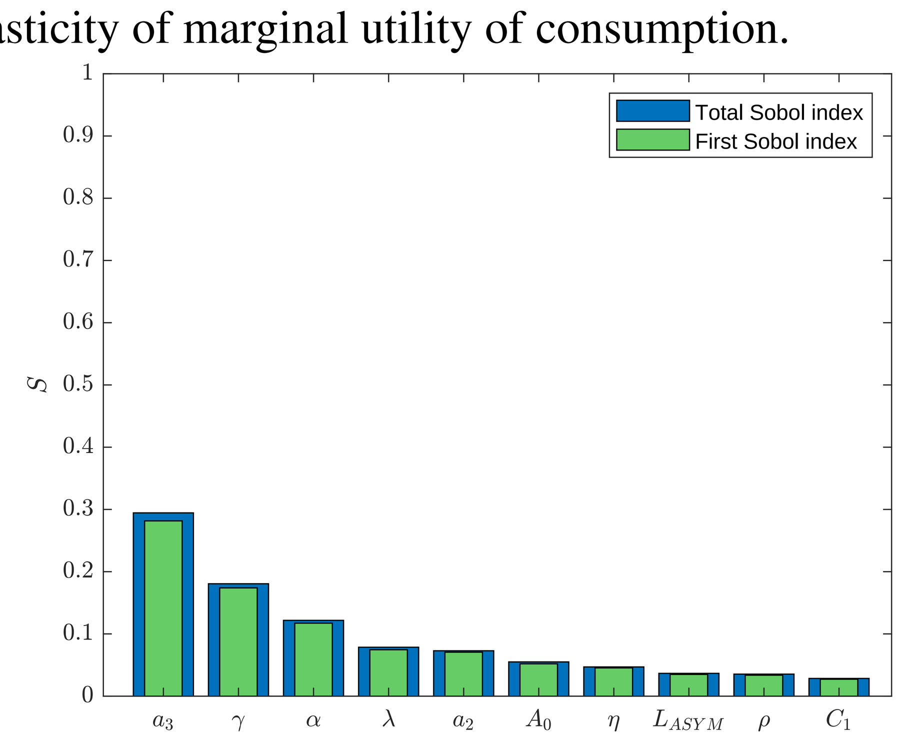

Figure 2: Total and first Sobol indices for 10 most important parameters.

Computational efficiency of PCE method

An important feature of PCE-based GSA is small number of model runs required for the decomposition. For the set of 8 parameters, PCE of degree 3 on a sample of size 200 provides good approximation (Figure 3).

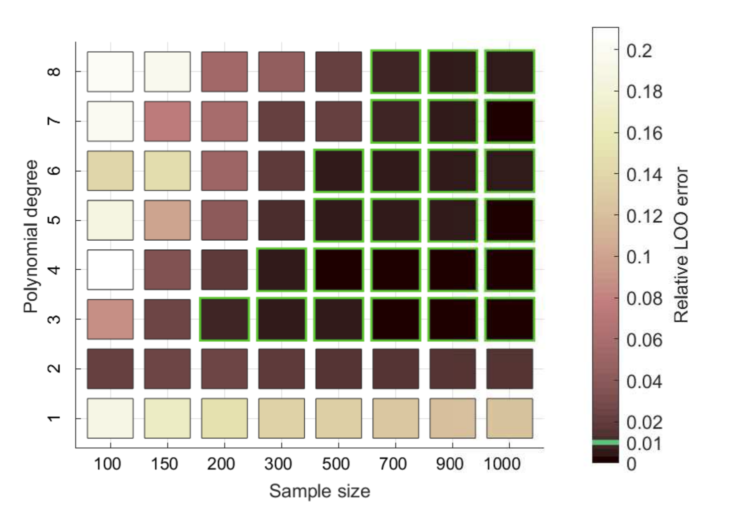

Figure 3: Relative approximation error for increasing sample sizes and polynomial degrees.

(In)dependence of parameters

Variance-based GSA relies on the *assumption of independence* of all parameters. *Spurious significance* of sensitivity indices is one result of misapplication of this assumption [4]. In DICE, some relationships are fundamental to the model and have to be respected.

1. **Ramsey rule.** $r^* = \rho + \alpha g^*$.
   Parameters ρ and α are jointly calibrated such that the *endogenous* r and g match target values.
2. **Initial state of economy.** $Y_0^{GROSS} = A_0 K_0^\gamma L_0^{1-\gamma}$.
   A_0 is determined once the other initial values and the value of γ are chosen.
3. **Damage function.** $\Omega_t = (1 + a_2 T_{A_t}^{a_3})^{-1}$.
   Parameters a_2 and a_3 are set such that Ω matches its benchmark values at particular levels of T_A.
4. **Carbon cycle.** $\mathbf{M_{t+1}} = \mathbf{\Phi M_t} + [E_t \ 0 \ 0]'$,
$$\mathbf{\Phi} = \begin{bmatrix} \phi_{11} & \phi_{21} & 0 \\ \phi_{12} & \phi_{22} & \phi_{32} \\ 0 & \phi_{23} & \phi_{33} \end{bmatrix}.$$
   Parameters ϕ_{12} and ϕ_{23} are calibrated to fit a benchmark impulse response.
5. **Initial state of the climate system.**
$$T_{AT_t} = \lambda \log\left(\frac{M_{AT_t}}{M_{AT}}\right) / \log(2).$$
   The initial levels M_{AT_0} and T_{AT_0}, and climate sensitivity λ have to keep this fundamental relationship.

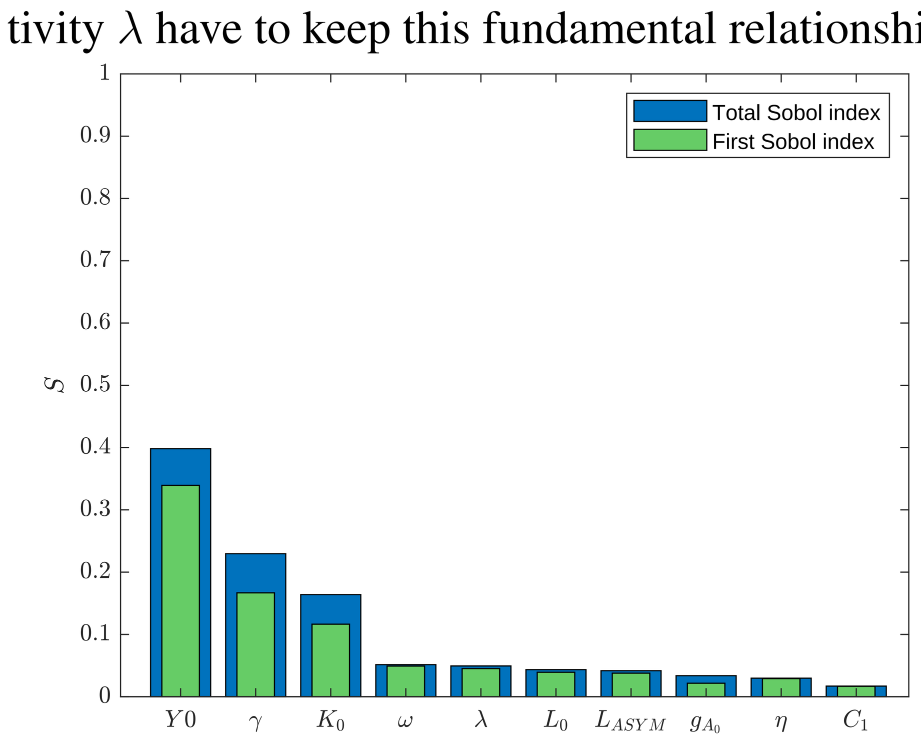

Figure 4: The largest 10 total and first Sobol indices in the setting where fundamental relatioships are preserved.

Conclusion

The methodology of efficient GSA provides a clear, comprehensive decomposition of the uncertainty in model's output while minimizing computational costs, and hence is easily applicable to more complex IAMs. When used in practice, the analysis should adjust to the the fundamental relations inherent in the model.

Parameter	Definition
α	Elasticity of marginal utility of consumption
γ	Capital elasticity in production function
η	Estimated forcings of equilibrium CO_2 doubling
λ	Equilibrium climate sensitivity
ρ	Discount rate
ϕ_{12}	Carbon cycle transition coefficient
ω	Damages at 3 degrees (relative to output)
a_2	Linear coefficient of damage function
a_3	Exponent of damage function
g_{A0}	Initial growth rate for technology
g_{σ_0}	Initial change of decarbonization
A_0	Initial level of total factor productivity
C_1	Climate-equation coefficient for upper level
$CCum$	Maximum extraction of fossil fuels
K_0	Initial level of capital
L_{ASYM}	Asymptotic population
P_{BACK}	Cost of backstop technology
Y_0	Initial level of output

References

[1] B. Anderson, E. Borgonovo, M. Galeotti, and R. Roson. Uncertainty in climate change modeling: Can global sensitivity analysis be of help? *Risk analysis : an official publication of the Society for Risk Analysis*, 34(2):271–293, 2014.

[2] S. Marelli and B. Sudret. UQLab: A framework for Uncertainty Quantification in MATLAB. 2014.

[3] W. D. Nordhaus. A Question of Balance: Weighing the Options on Global Warming Policies. 2008.

[4] A. Saltelli and S. Tarantola. On the Relative Importance of Input Factors in Mathematical Models. *Journal of the American Statistical Association*, 97(459):702–709, 2002.

[5] B. Sudret. Global sensitivity analysis using polynomial chaos expansions. *Reliability Engineering & System Safety*, 93(7):964–979, 2008.